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A non-central feeding hydrostatic thrust bearing

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In a conventional hydrostatic thrust bearing, the lubricant is supplied from the centre and flows radially outwards. It has been found that the load capacity of such a bearing decreases with increasing angular speed of the rotor. The bearing fails when a critical rotor speed is reached at which the load capacity becomes zero. In this paper a modified feeding system is suggested in which the lubricant is supplied from a ring-shaped groove situated between the exits at the centre and the edge of the bearing. Analysis shows that, by reversing the flow direction near the centre, the load capacity of a bearing of proper geometry can be made to decrease at a much slower rate than that of a conventional bearing as the rotor speed increases. The non-central feeding system might be considered for use in high-speed thrust bearings.

1. Introduction

In a conventional hydrostatic thrust bearing, shown in figure 1, lubricant is supplied at a pressure P_0 and flows radially outwards from a central plenum recess to the atmosphere, at an exhaust pressure P_e, through a thin gap of thickness $2h$. The load W applied on the upper disk is supported by the excess pressure $P-P_e$ in the lubrication film. When the upper disk rotates at a constant speed ω, viscous forces cause the fluid to move with the disk and a fluid particle follows a spiral path through the gap. Taking into consideration the fluid inertia due to its circumferential motion, Osterle & Hughes (1958) found that disk rotation reduces the load capacity of the bearing by an amount proportional to the dimensionless parameter $\rho b^2\omega^2/(P_0-P_e)$, where ρ is the lubricant density and b the radius of the bearing disks. Significant reduction in load capacity may result even at normal operating rotor speeds if a heavy lubricant is used or the pumping pressure is not much higher than the atmospheric pressure. This reduction increases quadratically with ω, and when a critical angular speed is reached the bearing can no longer carry any load.

In the present work it is shown that the decrease in load capacity due to disk rotation can be greatly reduced by modifying the lubricant feeding system. In the modified design shown in figure 2, a ring-shaped groove is added on the stationary lower disk. The radii of the inner and outer edges of the groove are respectively βb and γb. The central recess, of radius αb, is now used as a lubricant outlet in addition to the one at the edge of the bearing. Both outlets are kept at the atmospheric pressure P_e. When lubricant is supplied at a pressure P_0 from the

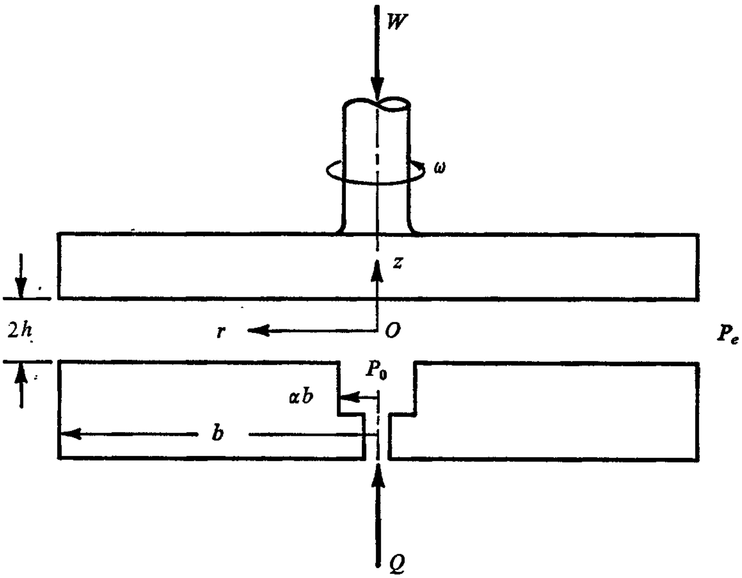

FIGURE 1. Conventional hydrostatic thrust bearing.

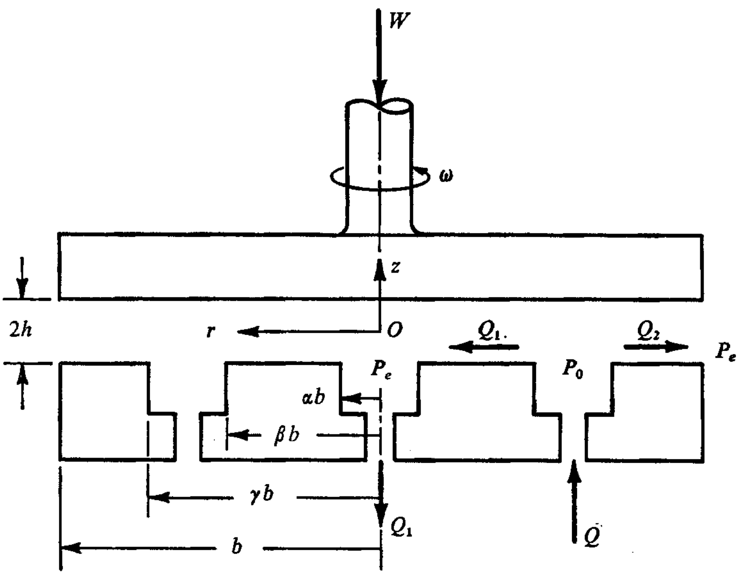

FIGURE 2. Non-central feeding hydrostatic thrust bearing.

groove, it divides itself into two streams, one flowing radially inwards and the other outwards. The two flow rates are controlled by the angular speed of the upper disk. In the following, the performance of this non-central feeding thrust bearing will be obtained and compared with that of a conventional bearing.

2. Bearing performance

In our analysis it is assumed that the flow is steady and symmetric about the bearing axis, that both the fluid density ρ and coefficient of viscosity μ are constant, that the thickness $2h$ of the lubrication film is very small compared with the bearing radius b, and finally, that the inertial force due to radial fluid motion is negligibly small compared with both that due to circumferential motion and the viscous forces. Under these assumptions and in cylindrical co-ordinates (r, θ, z), the equations of motion for fluid in the lubrication film are

$$\mu\,\partial^2 u/\partial z^2 + \rho\frac{v^2}{r} - \frac{\partial P}{\partial r} = 0, \tag{1}$$

$$\mu\,\partial^2 v/\partial z^2 = 0, \quad \partial P/\partial z = 0, \tag*{(2), (3)}$$

where u and v are respectively the velocity components in the r and θ directions.

The solution of (2) satisfying the boundary conditions $v = 0$ at $z = -h$ and $v = r\omega$ at $z = h$ is

$$v = \tfrac{1}{2}r\omega(1+z/h). \tag{4}$$

Equation (3) shows that the pressure is a function of r only, therefore the term $\partial P/\partial r$ in (1) can be replaced by an ordinary differential. By substituting v from (4), (1) becomes

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu}\frac{dP}{dr} - \frac{\rho r\omega^2}{4\mu}\left(1+\frac{z}{h}\right)^2, \tag{5}$$

whose solution is

$$u = -\frac{h^2}{2\mu}\frac{dP}{dr}\left[1-\left(\frac{z}{h}\right)^2\right] + \frac{\rho r h^2\omega^2}{6\mu}\left[\left(1+\frac{z}{h}\right) - \frac{1}{8}\left(1+\frac{z}{h}\right)^4\right], \tag{6}$$

which satisfies the boundary conditions $u = 0$ at $z = \pm h$.

Let the volumetric flow rate of the lubricant flowing towards the central outlet be Q_1. In the region $\alpha b \leqslant r \leqslant \beta b$ the continuity equation has the form

$$Q_1 = -\int_{-h}^{h} 2\pi r u\, dz. \tag{7}$$

Upon substitution of u from (6), it becomes

$$Q_1 = \frac{4\pi h^3 r}{3\mu}\frac{dP}{dr} - \frac{2\pi\rho r^2 h^3\omega^2}{5\mu}. \tag{8}$$

Since Q_1 is a constant, (8) can be integrated from $r = \alpha b$, where the pressure is P_e, to $r = \beta b$, where the pressure is P_0. The result shows that

$$Q_1 = Q_0\left[1 - \frac{3}{20}\frac{\rho b^2\omega^2}{P_0 - P_e}(\beta^2-\alpha^2)\right]\ln\frac{1}{\alpha}\bigg/\ln\frac{\beta}{\alpha}, \tag{9}$$

where

$$Q_0 = 4\pi h^3(P_0 - P_e)/3\mu\ln\alpha^{-1} \tag{10}$$

is the well-known flow rate in a conventional thrust bearing with stationary disks.

Let Q_2 denote the volumetric flow rate of the lubricant flowing towards the edge of the bearing. Then in the region $\gamma b \leqslant r \leqslant b$

$$Q_2 = \int_{-h}^{h} 2\pi r u \, dz, \tag{11}$$

and after the integration has been performed with the help of (6),

$$Q_2 = -\frac{4\pi h^3 r}{3\mu}\frac{dP}{dr} + \frac{2\pi\rho r^2 h^3 \omega^2}{5\mu}. \tag{12}$$

Integrating (12) from $r = \gamma b$, where the pressure is P_0, to $r = b$, where the pressure is P_e, we obtain

$$Q_2 = Q_0\left[1 + \frac{3}{20}\frac{\rho b^2\omega^2}{P_0 - P_e}(1-\gamma^2)\right]\ln\frac{1}{\alpha}\Big/\ln\frac{1}{\gamma}. \tag{13}$$

Equations (9) and (13) show that the rotational motion of the upper disk decreases the inward flow rate Q_1 while it increases the outward flow rate Q_2, which can be explained by the fact that the centrifugal acceleration caused by the circumferential fluid motion is always in the outward radial direction. The total flow rate in the non-central feeding bearing can be computed from the relationship

$$Q = Q_1 + Q_2. \tag{14}$$

We denote the flow rate in a conventional thrust bearing by Q_c, the expression for which can be obtained by replacing γ by α on the right-hand side of (13):

$$Q_c = Q_0\left[1 + \frac{3}{20}\frac{\rho b^2\omega^2}{P_0 - P_e}(1-\alpha^2)\right]. \tag{15}$$

A comparison of Q/Q_0 and Q_c/Q_0 is shown in figure 3 for the values $\alpha = 0{\cdot}1$, $\beta = 0{\cdot}5$ and $\gamma = 0{\cdot}6$. For the same film thickness and pressure difference, the flow rate in a non-central feeding bearing is higher because of the higher pressure gradients in the lubrication film, and increases more rapidly with ω than does that in a conventional bearing.

The pressure distribution can be obtained by integrating (8) and (12) from $r = \alpha b$ and γb, respectively, to an arbitrary radius r within their respective regions of validity. Thus we obtain for $\alpha b \leqslant r \leqslant \beta b$

$$P - P_e = (P_0 - P_e)\ln\frac{r}{\alpha b}\Big/\ln\frac{\beta}{\alpha} - \tfrac{3}{20}\rho b^2\omega^2\left[(\beta^2-\alpha^2)\ln\frac{r}{\alpha b}\Big/\ln\frac{\beta}{\alpha} - \left(\frac{r^2}{b^2}-\alpha^2\right)\right], \tag{16}$$

and for $\gamma b \leqslant r \leqslant b$

$$P - P_e = (P_0 - P_e)\ln\frac{r}{\gamma b}\Big/\ln\frac{1}{\gamma} + \tfrac{3}{20}\rho b^2\omega^2\left[(1-\gamma^2)\ln\frac{r}{\gamma b}\Big/\ln\frac{1}{\gamma} - \left(\frac{r^2}{b^2}-\gamma^2\right)\right]. \tag{17}$$

The pressure distribution in a non-central feeding bearing is sketched in figure 4 in comparison with that in a conventional one, which is obtained by replacing γ in (17) by α.

The load W which a bearing can carry is equal to the total force acting on the upper disk owing to the excess pressure $P - P_e$. For a non-central feeding bearing, according to figure 4,

$$W = \int_{\alpha b}^{\beta b} 2(P-P_e)\pi r\, dr + \pi b^2(1-\beta^2)(P_0-P_e) - \int_{\gamma b}^{b} 2(P_0-P)\pi r\, dr. \tag{18}$$

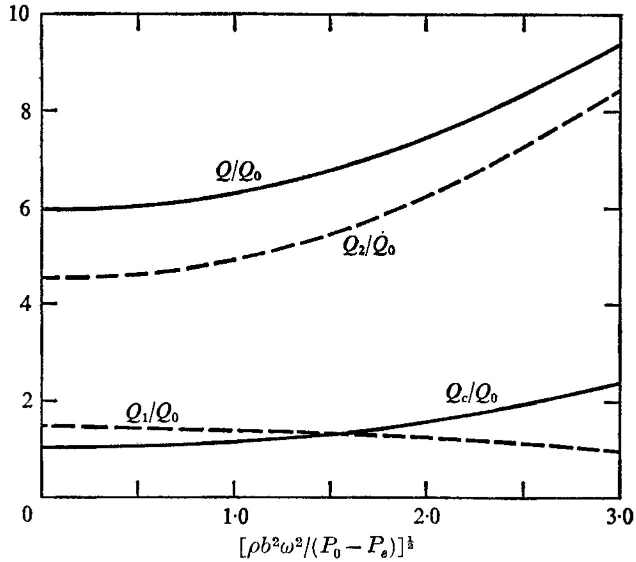

FIGURE 3. Comparison of Q/Q_0 and Q_c/Q_0 at various rotor speeds for $\alpha = 0{\cdot}1$, $\beta = 0{\cdot}5$ and $\gamma = 0{\cdot}6$.

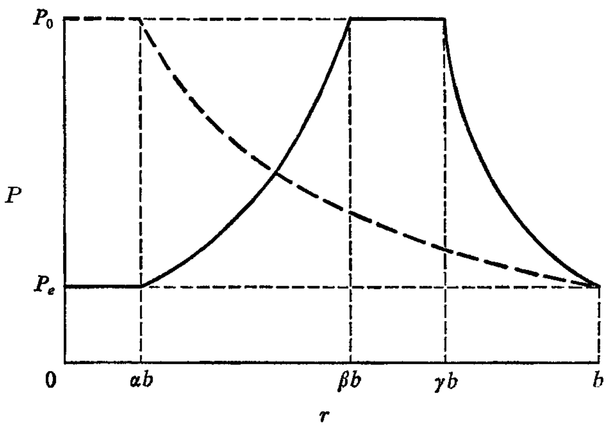

FIGURE 4. Sketch of pressure distributions for a conventional bearing (dashed curve) and a non-central feeding bearing (solid curve).

Substituting (16) into the first integral and (17) into the second integral in (18), we obtain

$$W = W_0\left[f_1(\alpha,\beta,\gamma)-\frac{3}{20}\frac{\rho b^2\omega^2}{P_0-P_e}f_2(\alpha,\beta,\gamma)\right]\frac{\ln\alpha^{-1}}{1-\alpha^2}, \tag{19}$$

where
$$f_1(\alpha,\beta,\gamma) = (1-\gamma^2)/\ln\gamma^{-1}-(\beta^2-\alpha^2)/\ln\beta\alpha^{-1} \tag{20}$$
and
$$f_2(\alpha,\beta,\gamma) = (1-\gamma^4)-(1-\gamma^2)^2/\ln\gamma^{-1}+(\beta^4-\alpha^4)-(\beta^2-\alpha^2)^2/\ln\beta\alpha^{-1} \tag{21}$$
are functions determined by the bearing geometry, and

$$W_0 = \tfrac{1}{2}\pi b^2(P_0-P_e)(1-\alpha^2)/\ln\alpha^{-1} \tag{22}$$

is the load capacity of a conventional bearing in the absence of disk rotation.

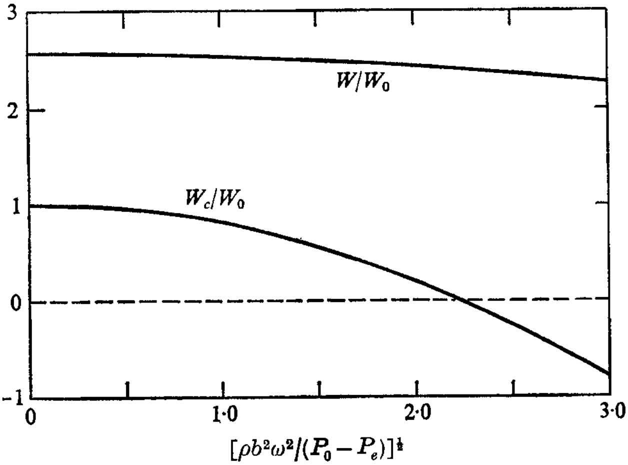

FIGURE 5. Comparison of W/W_0 and W_c/W_0 at various rotor speeds for $\alpha = 0{\cdot}1$, $\beta = 0{\cdot}5$ and $\gamma = 0{\cdot}6$.

On the other hand, the load capacity of a conventional bearing having a rotating disk is

$$W_c = \pi b^2(P_0-P_e) - \int_{\alpha b}^{b} 2(P_0-P)\,\pi r\,dr$$

$$= W_0\left\{1-\frac{3}{20}\frac{\rho b^2\omega^2}{P_0-P_e}\left[(1+\alpha^2)\ln\frac{1}{\alpha}-(1-\alpha^2)\right]\right\}, \tag{23}$$

which is equivalent to the result obtained by Osterle & Hughes.

Figure 5 shows a comparison of W/W_0 and W_c/W_0 at different angular speeds for $\alpha = 0{\cdot}1$, $\beta = 0{\cdot}5$ and $\gamma = 0{\cdot}6$. For a stationary upper disk, the load capacity of a non-central feeding bearing is higher than that of a conventional one because of the larger area of the high pressure region in the new design. A special characteristic of the non-central feeding bearing is that with increasing ω its load capacity decreases at a much slower rate than that of a conventional bearing. For instance, at $[\rho b^2\omega^2/(P_0-P_e)]^{\frac{1}{2}} = 2{\cdot}24$, W_c becomes zero while the reduction in W is only about 5·6 %.

Equations (19) and (23) indicate that a critical angular speed ω_{cr} depending upon the geometry of the stationary lower disk exists for a bearing at which it cannot carry any load. The non-dimensionalized critical values for a non-central feeding and a conventional thrust bearing are given, respectively, by

$$\rho b^2\omega_{cr}^2/(P_0-P_e) = \tfrac{20}{3}f_1(\alpha,\beta,\gamma)/f_2(\alpha,\beta,\gamma) \tag{24}$$

and

$$\rho b^2(\omega_{cr})_c^2/(P_0-P_e) = \tfrac{20}{3}/[(1+\alpha^2)\ln\alpha^{-1}-(1-\alpha^2)]. \tag{25}$$

For $\alpha = 0{\cdot}1$, $\beta = 0{\cdot}5$ and $\gamma = 0{\cdot}6$, the ratio $\omega_{cr}/(\omega_{cr})_c$ of the critical angular speeds is 3·94.

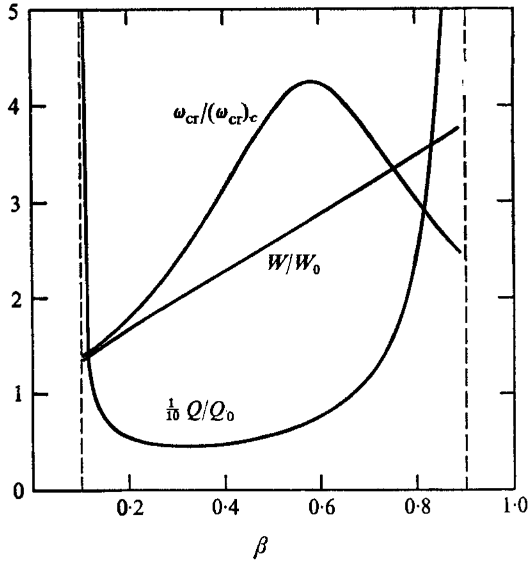

FIGURE 6. Performance of a non-central feeding bearing as a function of β for $\alpha = 0{\cdot}1$ and $\gamma - \beta = 0{\cdot}1$. Q and W were computed for $\omega = 0$.

3. A numerical example

The characteristics of a non-central feeding thrust bearing are functions of the radius of the central recess, the width of the ring-shaped groove and its location. A series of curves describing the bearing's performance can be plotted by fixing two variables and varying the third. As an illustrative example, we consider a bearing in which the radius of the central outlet and the groove width are both one-tenth of the disk radius, i.e. $\alpha = 0{\cdot}1$ and $\gamma - \beta = 0{\cdot}1$, and plot in figure 6 the bearing's performance as a function of β, which is the dimensionless distance of the inner edge of the groove from the axis. For stationary bearing disks, the dimensionless load capacity W/W_0 increases continuously with β because the area covered by the high pressure region increases. The dimensionless flow rate Q/Q_0 has a minimum value of 4·61 at $\beta = 0{\cdot}30$ and increases without bound when β approaches either 0·1 or 0·9. This phenomenon is explained by the fact that, when the lubricant inlet is approaching an outlet, the retarding viscous force within the region in between decreases while the pressure gradient there increases, so that an extremely high flow rate is produced by the large net resultant force. For a rotating upper disk, both the flow rate and load capacity are functions of the angular speed for a specified value of β. The curves of Q/Q_0 in figure 3 and W/W_0 in figure 5 form one example for $\beta = 0{\cdot}5$. The ratio of the critical speed of the non-central feeding bearing to that of a conventional bearing, calculated from (24) and (25), is also plotted in figure 6 as a function of β. This ratio is always greater than unity and reaches a maximum value of 4·25 at $\beta = 0{\cdot}58$.

More specifically, let us examine a typical hydrostatic thrust bearing of outer radius 3 in. using a conventional feeding system. A lubricant oil of specific gravity of 0·9 is pumped through a central recess of radius 0·3 in. at a pressure of 200 psi over the atmospheric value. According to (25) the critical angular speed of such a bearing is 10 948 r.p.m. Suppose that the operating conditions of the bearing are changed such that the limiting speed becomes doubled. This requirement can be met either by reducing the outer radius of the bearing to 1·8 in., or by increasing the pressure difference to 800 psi, or by a proper combination of changes in both radius and pressure. However, such modifications involve changes in the load density of the bearing and/or in the size of the pump, which may result in a major revision of the entire design. To avoid this trouble, one of the solutions might be to change from the conventional to the non-central feeding system. Keeping the same bearing dimensions, if the lubricant is pumped in at the same pressure from a groove 0·3 in. wide whose inner edge is at a radial distance of 0·9 in. from the axis, the critical angular speed of the bearing is increased to 25 576 r.p.m. according to (24). The critical speed can be increased further, to 46 398 r.p.m., if the inner edge of the 0·3 in. groove is moved outwards to 1·8 in. from the axis. On the other hand, in order for a conventional bearing of the same size to reach a limiting speed of 46 398 r.p.m., the pressure difference has to be raised to more than 16 times the original value. The advantage of using the non-central feeding system to achieve high angular speeds for a hydrostatic thrust bearing is apparent.

However, changing to a non-central feeding system is by no means the only method of solving such an engineering problem. For instance, by replacing the hydrostatic by a hydrodynamic thrust bearing, the angular-speed limitation is removed. The present work merely suggests a possible improvement on the conventional hydrostatic bearings. Furthermore, for practical applications the stability characteristics of bearings equipped with central and non-central feeding systems must be computed and compared. The feasibility of the proposed feeding system will be determined by further theoretical and experimental investigations.

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